

A hypothesis on neutrino helicity

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Abstract

It is firmly established by experimental results that neutrinos are almost 100% longitudinally polarized and left-handed. It is also confirmed by neutrino oscillation experiments that neutrinos have tiny but non-zero masses. Since their masses are non-zero the neutrinos cannot be strictly described by pure helicity states which coincide with the chirality eigenstates. On the other hand, it is generally assumed that ultrarelativistic massive fermions can be described well enough by the Weyl equations. This assumption obviously explains why the neutrinos are almost 100% longitudinally polarized. We discuss the validity of this assumption and show that the assumption is fallacious for a fermion with a general spin orientation. For instance, a fermion with a transverse polarization (relative to its momentum) cannot be described by one of the Weyl equations even in the ultrarelativistic limit. Hence, the fact that neutrinos are almost completely longitudinally polarized cannot be explained in the basis of relativistic quantum mechanics or quantum field theory. As a solution to this problem, we propose a new hypothesis according to which neutrinos are strictly described by pure helicity states although they are not massless.

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I. INTRODUCTION

In the original version of the Standard Model of particle physics neutrinos are accepted to be massless. Hence, they are described by pure helicity states which is consistent with the results obtained from experiments [1–5]. On the other hand, neutrino oscillation experiments point out an extension of the Standard Model. In the minimal extension of the Standard Model with massive neutrinos, flavor and mass eigenstates do not coincide. Flavor eigenstates can be written as a superposition of mass eigenstates through the mixing equation $\nu_{\ell L} = \sum_{i=1}^3 U_{\ell i} \nu_{iL}$ where $U_{\ell i}$ is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix element [6]. Since a flavor eigenstate is a mixture of different mass eigenstates each has a different rest frame, the rest frame of a flavor neutrino is somewhat uncertain.¹ However, the notion of spin is closely related to the rest frame of the particle. The spin four-vector for a fermion

$$s^\mu = \left(\frac{\vec{p} \cdot \vec{s}'}{m}, \vec{s}' + \frac{\vec{p} \cdot \vec{s}'}{m(E + m)} \vec{p} \right) \quad (1)$$

is obtained by a Lorentz boost of $(s^\mu)_{RF} = (0, \vec{s}')$ from the rest frame of the particle [7]. Therefore it is reasonable to accept that the spin state for a flavor neutrino is not well-defined. As far as we know, this problem has been skipped in the literature possibly because neutrino masses are extremely small ($m_{1,2,3} \lesssim 1 \text{ eV}$), and hence it is a very good approximation to use expressions obtained in the limit $m_{\nu_i} \rightarrow 0$. It is generally believed that free solutions of the Dirac equation coincide with pure helicity states (which correspond to chirality eigenstates) in the zero-mass limit.² Therefore one can assume that each of the

¹ In this paper only Dirac neutrinos have been considered. According to Dirac equation the states of definite momentum are not eigenstates of velocity. Hence, one may argue that the rest frame for a massive fermion is always uncertain. On the other hand, if we perform a time average over the period of Zitterbewegung (which is an extremely small time period) we obtain the classical velocity $\frac{c^2 \vec{p}}{E}$. In the case of flavor neutrinos we have an additional uncertainty in the rest frame due to neutrino mixing and this uncertainty cannot be removed even though an average over the period of Zitterbewegung is performed.

² We should clarify the terminology used in this paper. Sometimes, when the spin three-vector of a fermion with non-zero mass is oriented parallel (anti-parallel) to the direction of momentum, the corresponding spinor is called right-handed (left-handed). But since you can convert a right-handed fermion into a left-handed one simply by changing your frame of reference, the helicity states defined in this way are not intrinsic to the particle. When we use the term left(right)-handed helicity for a fermion with non-zero mass we imply the above meaning of the helicity. On the other hand, when we use the term *pure helicity states* we imply Lorentz invariant helicity for a massless fermion which correspond to chirality eigenstates.

constituent mass eigenstates for a flavor neutrino is described well enough by a pure helicity state. It is natural to define the spin of a flavor neutrino in terms of the spin of its constituent mass eigenstates. Hence, one may conceive the spin state for a flavor neutrino as a superposition of the spin states of different mass eigenstates. Since the spin of all mass eigenstates are equal to each other with a high accuracy (they are all approximately left-handed) we can uniquely define the spin of the flavor eigenstate. Here, we should be aware of the approximation that is used in the definition of the spin for a flavor eigenstate. Strictly speaking, the notion of spin and its special orientation helicity (see footnote 2) is not uniquely defined for a flavor neutrino. The assumption that a flavor neutrino has a uniquely defined spin state that coincides with left-handed helicity is an approximation which is valid with some degree of accuracy. This approximation is based on the assumption that ultrarelativistic massive fermions can be described well enough by the Weyl equations. According to this assumption, the free solutions of the Dirac equation containing a very small mass term (compared to the energy scale that we consider) can be described by pure helicity states with a high degree of accuracy. On the contrary, in the next section we will discuss some counter evidences obtained from spin dependent neutrino cross section and relativistic quantum mechanics that show this assumption is not valid in general. In the absence of this assumption, we encounter some serious problems. We cannot define the spin state of a flavor neutrino uniquely. We can only conceive the flavor spin as a superposition of the spin states of its constituent mass eigenstates where each spin state may have an arbitrary spin orientation. This result obviously contradicts with firmly established experimental results that show flavor neutrinos are almost 100% longitudinally polarized and left-handed. We call this problem the neutrino helicity problem. Our new hypothesis is an attempt which is proposed as a solution to the neutrino helicity problem.

The organization of the paper is as follows. In the next section first we will introduce our new hypothesis and then discuss some evidences obtained from spin dependent cross section and relativistic quantum mechanics that show generally believed assumption discussed in the previous paragraph is not valid in general. Absence of this assumption leads to the neutrino helicity problem and forces us to assert a new hypothesis. In the conclusion section we will discuss the implications of the new hypothesis on the foundations of physics.

Of course, two different meaning of the helicity coincide in the $m \rightarrow 0$ limit.

II. A HYPOTHESIS ON NEUTRINO HELICITY AND SOME EVIDENCES

In his famous 1939 paper Wigner investigated unitary representations of the Poincaré group and classified particles according to their internal space-time symmetries [8]. One of the important criteria used in Wigner's classification is the existence of the rest frame of a particle. There is no frame of reference in which a massless particle such as a photon is at rest. Hence the little group for a massless particle is $E(2)$ -like. On the other hand, a massive particle has a rest frame and in this frame we can rotate its spin three-vector without changing the momentum. Its little group is then $O(3)$ -like. We assert the following hypothesis which makes a flavor neutrino 100% longitudinally polarized: The neutrino flavor eigenstate is a mixture of different mass eigenstates each has a different rest frame. Hence, the rest frame of a flavor neutrino is uncertain and this uncertainty makes its rest frame undefined. Since there is no frame of reference in which a flavor neutrino is at rest, its little group is no more $O(3)$ -like. It should be classified together with massless particles described by $E(2)$ -like little group, although it has a non-zero mass and does not propagate at the speed of light. As we have discussed in the introduction, one may interpret the spin state for a flavor neutrino as a superposition of the spin states of different mass eigenstates. We think that this interpretation is not correct. Our interpretation is the following: since the definition of spin requires the existence of the rest frame and the rest frame of a flavor neutrino is uncertain, we do not have a well-defined spin state for a flavor neutrino. On the other hand, the definition of helicity (we mean a Lorentz invariant helicity which coincides with chirality. Please see footnote 2.) does not require the existence of the rest frame. Consequently, the helicity of the flavor neutrino is a well-defined quantity.

A. Evidences From Spin Dependent Cross section

It is generally assumed that ultrarelativistic massive fermions can be described well enough by the Weyl equations. Hence neutrinos are accepted to be completely longitudinally polarized and pure helicity states for the neutrino fields are used in the cross section calculations. Let us perform cross section calculations for neutrinos with a general spin orientation and probe the validity of this assumption. We will consider a simple particular process, namely polarized neutrino production via electron capture, where much of the com-

putation can be done easily. This process can be written at the quark level as $e^-u \rightarrow \nu_i d$, where ν_i represents a neutrino in the mass eigenstate. The process $e^-u \rightarrow \nu_i d$ is described by a t-channel W exchange diagram. Spin dependent amplitude for the process is given by³

$$M = \frac{G_F}{\sqrt{2}} U_{ei} U_{ud} \left[\bar{u}(p_{\nu_i}, s'_{\nu_i}) \hat{\Sigma}(s_{\nu_i}) \gamma^\mu (1 - \gamma_5) u(p_e, s'_e) \right] [\bar{u}(p_d, s'_d) \gamma_\mu (1 - \gamma_5) u(p_u, s'_u)] \quad (2)$$

where G_F is the Fermi constant, U_{ei} is the PMNS matrix element, U_{ud} is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element and $\hat{\Sigma}(s_{\nu_i}) \equiv \frac{1}{2}(1 + \gamma_5 \gamma_\mu s_{\nu_i}^\mu)$ is the covariant spin projection operator for the neutrino. In the rest frame of the neutrino the spin four-vector is $(s_{\nu_i}^\mu)_{RF} = (0, \vec{s}_{\nu_i})$. In an arbitrary reference frame the spin four-vector can be obtained by a Lorentz boost from the rest frame. In a reference frame where the neutrino has a momentum \vec{p} and energy E its spin four-vector can be defined by Eq.(1) with $m = m_{\nu_i}$ and $\vec{s}' = \vec{s}_{\nu_i}$. When we square the amplitude and sum over fermion spins the projection $\hat{\Sigma}(s_{\nu_i}) u(p_{\nu_i}, s'_{\nu_i}) = \delta_{s'_{\nu_i}, s_{\nu_i}} u(p_{\nu_i}, s_{\nu_i})$ ensures that the sum over s'_{ν_i} yields just one term with $s'_{\nu_i} = s_{\nu_i}$ [7]. The spin-summed squared amplitude is calculated to be

$$\sum_{s'_{\nu_i}, s'_e, s'_d, s'_u} |M|^2 = 64 G_F^2 |U_{ei}|^2 |U_{ud}|^2 [(p_e \cdot p_u)(p_{\nu_i} \cdot p_d) - m_{\nu_i}(p_e \cdot p_u)(s_{\nu_i} \cdot p_d)]. \quad (3)$$

It describes polarized neutrinos with spin four-vector s_{ν_i} but unpolarized electrons, u and d quarks. At first glance, it seems as if spin dependent term in Eq.(3) vanishes in the $m_{\nu_i} \rightarrow 0$ limit. But spin four-vector contains terms inversely proportional to m_{ν_i} . Therefore first we should perform Lorentz scalar products and then examine its zero-mass limit. After Lorentz scalar products are performed, the spin-summed squared amplitude can be written as

$$\begin{aligned} \sum_{s'_{\nu_i}, s'_e, s'_d, s'_u} |M|^2 = & 64 G_F^2 |U_{ei}|^2 |U_{ud}|^2 (E_e E_u - \vec{p}_e \cdot \vec{p}_u)(E_{\nu_i} E_d - E_d(\vec{s}_{\nu_i} \cdot \vec{p}_{\nu_i}) \\ & + \vec{p}_{\nu_i} \cdot \vec{p}_d \left(\frac{\vec{s}_{\nu_i} \cdot \vec{p}_{\nu_i}}{E_{\nu_i} + m_{\nu_i}} - 1 \right) + m_{\nu_i} \vec{s}_{\nu_i} \cdot \vec{p}_d). \end{aligned} \quad (4)$$

We observe from Eq.(4) that the term $\vec{s}_{\nu_i} \cdot \vec{p}_{\nu_i}$ does not completely vanish in the $m_{\nu_i} \rightarrow 0$ limit. Therefore it doesn't matter how small it is, if the neutrino has a nonzero mass then the cross section depends on its spin orientation. In the center-of-momentum frame zero-mass

³ We assume that the W propagator can be approximated as $\frac{(g_{\mu\nu} - q_\mu q_\nu / m_w^2)}{q^2 - m_w^2} \approx -\frac{g_{\mu\nu}}{m_w^2}$.

limit of the squared amplitude becomes

$$\lim_{m_{\nu_i} \rightarrow 0} \sum_{s'_{\nu_i}, s'_e, s'_d, s'_u} |M|^2 = 64G_F^2 |U_{ei}|^2 |U_{ud}|^2 (E_e E_u + |\vec{p}_e| |\vec{p}_u|) \times (E_d + |\vec{p}_d|) (E_{\nu_i} - \vec{s}_{\nu_i} \cdot \vec{p}_{\nu_i}). \quad (5)$$

We see from the above expression that the squared amplitude and hence the cross section takes its largest value when the neutrino is left-handed ($\vec{s}_{\nu_i} = -\frac{\vec{p}_{\nu_i}}{|\vec{p}_{\nu_i}|}$) and zero when the neutrino is right-handed ($\vec{s}_{\nu_i} = +\frac{\vec{p}_{\nu_i}}{|\vec{p}_{\nu_i}|}$). If the spin three-vector is perpendicular to the direction of neutrino momentum then the cross section is half of the cross section for left-handed neutrino. In general, if the spin three-vector makes an angle ϕ with respect to direction of neutrino momentum then we deduce that the zero-mass limits of the cross sections in the center-of-momentum frame for spin up and spin down polarizations are given by

$$\sigma^{(\uparrow)}(s_{\nu_i}) = \sin^2\left(\frac{\phi}{2}\right) \sigma^{(L)} \quad (6)$$

$$\sigma^{(\downarrow)}(s_{\nu_i}) = \cos^2\left(\frac{\phi}{2}\right) \sigma^{(L)} \quad (7)$$

where $\sigma^{(L)}$ represents cross section for left-handed neutrino. (Of course, unpolarized total cross section remains unchanged, i.e., $\sigma_{unpol} = \sigma^{(\uparrow)} + \sigma^{(\downarrow)} = [\sin^2(\frac{\phi}{2}) + \cos^2(\frac{\phi}{2})] \sigma^{(L)} = \sigma^{(L)}$) In the above equations the limit $m_{\nu_i} \rightarrow 0$ is implemented but not shown. We should note that spin down polarization corresponds to a spin three-vector which is oriented opposite to the direction of spin three-vector for spin up, i.e., spin three-vector for spin down polarization makes an angle $\phi + \pi$ with respect to direction of momentum. In opposition to expectations these results indicate that the transverse component of the spin three-vector of a fermion does not vanish in the zero-mass limit.⁴ This result contradicts with the generally accepted assumption that when the speed of a particle approaches the speed of light its spin vector lays down on the momentum direction. This assumption is obviously related to the assumption mentioned in the introduction section which says free solutions of the Dirac equation coincide with pure helicity states in the zero-mass limit. Is it indeed possible that this generally accepted assumption is fallacious? We should examine what the relativistic

⁴ When we use the notation $m \rightarrow 0$ or the phrase "zero-mass limit" we mean an infinitesimal mass but not equal to zero.

quantum mechanics says about the zero-mass limit of a spinor describing a general spin orientation.

B. Evidences From Relativistic Quantum Mechanics

Let us construct the spinors describing a general spin orientation and examine their behavior in the zero-mass limit. Assume that S defined by spatial axes x - y - z and time t is the rest frame of a spin-1/2 particle with mass m . In the rest frame of the particle we can safely use Pauli spinors to define its spin. Let q -axis be the spin quantization axis in the z - x plane which makes an angle ϕ with respect to z -axis. Then the non-relativistic 2×2 spin matrix is $\hat{S} = \frac{1}{2}(\sin \phi \hat{\sigma}_x + \cos \phi \hat{\sigma}_z)$ where $\hat{\sigma}_x, \hat{\sigma}_y$ and $\hat{\sigma}_z$ are Pauli spin matrices. The eigenvectors of the spin matrix are

$$\chi_+ = \begin{pmatrix} \cos \phi/2 \\ \sin \phi/2 \end{pmatrix} \quad \chi_- = \begin{pmatrix} -\sin \phi/2 \\ \cos \phi/2 \end{pmatrix}. \quad (8)$$

Here χ_+ and χ_- are the eigenvectors which correspond to eigenvalues $\lambda = +1$ and $\lambda = -1$ respectively, i.e., $\hat{S}\chi_+ = \chi_+$ and $\hat{S}\chi_- = -\chi_-$. We have so far considered only 2×1 matrix representations. In the Weyl representation, 4×1 Dirac spinors for a spin-1/2 particle can be written as [9]

$$u(p) = \begin{pmatrix} \phi_R(p) \\ \phi_L(p) \end{pmatrix} \quad (9)$$

where $\phi_R(p)$ and $\phi_L(p)$ are 2×1 spinors and subscripts "R" and "L" represents chirality. In the rest frame of the particle the equality $\phi_R(0) = \phi_L(0)$ holds. $\phi_R(0)$ and $\phi_L(0)$ can be considered as the eigenvectors of the 2×2 spin matrix. The spinor $u(p)$ for a particle with four-momentum $p^\mu = (E, \vec{p})$ can be obtained via Lorentz boost from rest spinor $u(0)$. Suppose that S' frame is moving along the negative z -axis with relative speed v with respect to S . If we choose $\phi_R(0) = \phi_L(0) = \chi_+$ and perform a Lorentz boost into S' frame then we obtain a Dirac spinor which describes a particle of four-momentum $p^\mu = (E, 0, 0, p_z)$ and four-spin given in Eq.(1) where $\vec{s} = \sin \phi \hat{x} + \cos \phi \hat{z}$ is the unit vector on the spin quantization axis q in the rest frame of the particle. The spinor obtained in this way describes a "spin up" state. Similarly if we choose $\phi_R(0) = \phi_L(0) = \chi_-$ and perform a Lorentz boost,

then we obtain the "spin down" spinor.⁵ After some straightforward calculations, these "spin up" (\uparrow) and "spin down" (\downarrow) spinors are found to be

$$u^{(\uparrow)}(p, s) = \cos\left(\frac{\phi}{2}\right) u^{(R)}(p, s_*) + \sin\left(\frac{\phi}{2}\right) u^{(L)}(p, s_*) \quad (10)$$

$$u^{(\downarrow)}(p, s) = \cos\left(\frac{\phi}{2}\right) u^{(L)}(p, s_*) - \sin\left(\frac{\phi}{2}\right) u^{(R)}(p, s_*) \quad (11)$$

where $u^{(R)}(p, s_*)$ and $u^{(L)}(p, s_*)$ represent right-handed and left-handed spinors, i.e., $\vec{s}_* = \lambda \frac{\vec{p}}{|\vec{p}|}$, $\lambda = +1(-1)$ for right-handed (left-handed). We observe from Eqs. (10) and (11) that "spin up" and "spin down" spinors can be written as a superposition of right- and left-handed spinors. We now come to a controversial point. One may assume that the angle ϕ converges to zero due to relativistic aberration when the relative speed approaches the speed of light. We claim that this assumption is fallacious because ϕ is the angle measured in the frame in which the particle is at rest. In analogy with the term proper time we can call it "proper angle". Then, coefficients $\cos(\frac{\phi}{2})$ and $\sin(\frac{\phi}{2})$ do not depend on the energy or the mass of the particle. Hence, these "spin up" and "spin down" spinors do not approach one type of helicity state (R or L) when the mass approaches zero. They are always given by the same superposition of right- and left-handed spinors. For the special case $\phi = \pi/2$, spin three-vector \vec{s} becomes perpendicular to the direction of momentum. In this case, $u^{(\uparrow)}(p, s)$ can be considered as a mixed state composed of helicity states where each helicity state has equal probability. (The similar thing is also true for $u^{(\downarrow)}(p, s)$.) This mixed state remains intact for every value of the mass greater than, but not equal to zero, i.e., $m > 0$. Since the helicity and chirality states coincide in the ultrarelativistic limit we can deduce that the spinors $u^{(\uparrow)}(p, s)$ and $u^{(\downarrow)}(p, s)$ do not converge to a pure helicity or a chirality state.

We conclude from the above discussion that although the helicity states converge to the chirality eigenstates when $m \rightarrow 0$, the free solutions of Dirac equation describing a general spin orientation *do not continuously* converge to chirality eigenstates. Here the phrase "*do not continuously*" is used to indicate the discontinuity at $m = 0$. We have shown that when the mass parameter is in the open interval $(0, \infty)$ general spin states and chirality eigenstates are disjointed. But in the point $m = 0$ the solution describing a general spin orientation jumps to a pure helicity state. This behavior seems contradictory to the fact that the

⁵ Hereafter we will use the notation $u(p, s)$ instead of $u(p)$ and sometimes use the superscripts \uparrow and \downarrow for spin up and spin down and the superscripts R and L for right-handed and left-handed.

$E(2)$ -like little group is the Lorentz-boosted $O(3)$ -like ($SU(2)$ -like) little group for massive particles in the zero-mass limit [10–12]. However, the group contraction is established by relating the generators of these groups and hence it points out a local isomorphism between $SU(2)$ and $E(2)$ in the limit $m \rightarrow 0$. A general spin orientation can be obtained by a *finite* rotation from the direction in which the Lorentz boost is performed. Therefore it is controversial to conclude from a local isomorphism that the transverse component of the spin three-vector vanishes in the zero-mass limit. One may observe from Eq.(1) that when the speed of a particle approaches the speed of light, spatial component of its spin four-vector lays down on the momentum direction. Such a relativistic aberration is indeed true but the spatial component of the spin four-vector does not represent the spin orientation of the moving fermion. The true spin three-vector should be the one which is attached on the moving frame and coincides with the spin quantization axis. Therefore it should be: $\vec{s} - \vec{v}t = \vec{s}' - \frac{\vec{p} \cdot \vec{s}'}{E(E+m)} \vec{p}$. It corresponds to an imaginary ruler on the moving frame that coincides with the spin quantization axis. It is obvious that a ruler which is oriented transverse to the direction of Lorentz boost remains unchanged after the Lorentz transformation.

Cross section calculations can also be done directly by inserting the corresponding explicit expressions for Dirac matrices and spinors. But of course in this case, it is not necessary to use spin projection operator $\hat{\Sigma}(s_{\nu_i})$. It can be omitted from the amplitude. The standard model neutrinos couple minimally to other standard model particles only through $V - A$ type vertex and hence the interaction project out the left chiral component of the field and, in the $m \rightarrow 0$ limit, the right component decouples completely. But we see from Eqs. (10) and (11) that the spinors describing a general spin orientation (specifically transverse polarization) cannot be separated into left and right chirality eigenstates even in the $m \rightarrow 0$ limit. Hence the left chirality projection operator $\hat{L} = \frac{1}{2}(1 - \gamma_5)$ annihilates only right-handed constituent of the spinor whereas its left-handed constituent remains intact. To be precise, if we apply left chirality projection operator to "spin-up" and "spin-down" spinors in Eqs. (10) and (11) we obtain

$$\hat{L}u^{(\uparrow)} = \sin\left(\frac{\phi}{2}\right) u^{(L)}, \quad \hat{L}u^{(\downarrow)} = \cos\left(\frac{\phi}{2}\right) u^{(L)} \quad (12)$$

in the $m \rightarrow 0$ limit. For $\phi = \frac{\pi}{2}$ (transverse polarization) these give $\hat{L}u^{(\uparrow)} = \hat{L}u^{(\downarrow)} = \frac{1}{\sqrt{2}}u^{(L)}$. It is easy to verify cross section formulas given in Eqs. (6) and (7) using identities in Eq.(12). Therefore, two different calculation techniques -the first technique is based on the use of

covariant spin projection operator $\hat{\Sigma}(s_{\nu_i})$ and the second one is the direct calculation using explicit expressions for Dirac matrices and spinors- give exactly the same results for the process $e^-u \rightarrow \nu_i d$ with a general neutrino spin, i.e., for an arbitrary ϕ value. Consequently, relativistic quantum mechanics verifies the results obtained from spin dependent cross section calculations. Moreover, with the help of Dirac spinors describing a general spin orientation we deduce that the behavior of the spin dependent cross section in the zero-mass limit is not peculiar to the particular process $e^-u \rightarrow \nu_i d$ instead, all standard model processes where neutrinos take part in the initial or final states exhibit such a behavior. This is evident from the identities given in Eq.(12) and the $V - A$ coupling of the standard model neutrinos.

C. Discussion

Both spin dependent cross section calculations and the results obtained from relativistic quantum mechanics provide convincing evidences against the generally accepted assumption that ultrarelativistic massive fermions can be described well enough by the Weyl equations. We think that the evidences are convincing enough to reject the assumption. We have deduced that the spinors describing a general spin orientation (specifically transverse polarization) cannot be separated into left and right chirality eigenstates even in the zero-mass limit. Hence, a fermion with a general spin orientation cannot be described by one of the Weyl equations even though it possesses an extremely small mass. Consequently, the polarized cross section for producing a neutrino mass eigenstate with a general spin orientation is not small. Specifically, if the neutrino is right-handed the polarized cross section is almost zero (it is strictly zero for $m_i \rightarrow 0$) but if the neutrino is transversely polarized (relative to its momentum) the polarized cross section is almost half of the cross section for left-handed neutrino (it is strictly half of the left-handed cross section for $m_i \rightarrow 0$). Therefore, the probability of producing a neutrino mass eigenstate with a spin orientation different from left-handed polarization is not small. Then we can conceive the flavor eigenstate as a superposition of the mass eigenstates where each mass eigenstate may have an arbitrary spin orientation which can be very different from left-handed polarization. On the contrary, experimental results confirm the fact that flavor neutrinos are almost 100% longitudinally polarized and left-handed. Therefore, we cannot explain these experimental results based solely on relativistic quantum mechanics or quantum field theory. This lack of explanation is

the neutrino helicity problem that we have mentioned in the introduction section. The generally accepted assumption that we have falsified provided a fake solution to this problem. Hence, a new assumption is necessary to explain experimental results. We think that the simplest solution to the neutrino helicity problem is the new hypothesis that we have proposed. Here, we should draw attention to the following point. Our hypothesis and also the generally accepted assumption that we have falsified, are not about the left-handedness of neutrinos. Our hypothesis is an attempt to explain why the flavor neutrinos are completely longitudinally polarized ,i.e., transverse polarization vanishes for flavor neutrinos. The fact that all neutrinos are left-handed but all anti-neutrinos are right-handed is out of the scope of the hypothesis.

III. CONCLUSIONS

Massive particles which do not have a rest frame were not considered in the Wigner's work [8]. On the other hand, quantum mechanics makes such a peculiar case possible. Based on the discussion about neutrino spin we conclude that a modification or an extension of the Wigner's work is necessary. This modification is probably related to a more deeper problem, which is the unification of quantum mechanics with special relativity. It is generally believed that the unification of quantum mechanics with special relativity has been completed. Their offspring is the quantum field theory. Nevertheless, a new hypothesis and its evidences discussed in this paper raise some doubt on the completeness of this unification.

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- [1] M. Goldhaber, L. Grodzins, and A. W. Sunyar, *Helicity of Neutrinos*, Phys. Rev. **109**, 1015 (1958).
 - [2] L. P. Roesch, V. L. Telegdi, P. Truttmann, A. Zehnder, L. Grenacs and L. Palffy, *Direct Measurement Of The Helicity Of The Muonic Neutrino*, Am. J. Phys. **50**, 931 (1982).
 - [3] W. Fetscher, *Helicity Of The Muon-neutrino In Pi+ Decay: A Comment On The Measurement Of P (mu) Xi Delta / Rho In Muon Decay*, Phys. Lett. B **140**, 117 (1984).
 - [4] A. Heister *et al.* [ALEPH Collaboration], *Measurement of the Michel parameters and the ν_τ helicity in τ lepton decays*, Eur. Phys. J. C **22**, 217 (2001).

- [5] K. Abe *et al.* [SLD Collaboration], *Measurement of the tau-neutrino helicity and the Michel parameters in polarized e^+e^- collisions*, Phys. Rev. Lett. **78**, 4691 (1997) [hep-ex/9701020].
- [6] S. Bilenky, *Introduction to the Physics of Massive and Mixed Neutrinos* (Springer, 2010).
- [7] W. Greiner and J. Reinhardt, *Quantum Electrodynamics*, (Springer, 1994).
- [8] E. P. Wigner, *On Unitary Representations of the Inhomogeneous Lorentz Group*, Ann. Math. **40**, 149 (1939).
- [9] L. H. Ryder, *Quantum Field Theory*, (Cambridge University Press, Second Ed. 1996).
- [10] Y. S. Kim and E. P. Wigner, *Cylindrical Group and Massless Particles*, J. Math. Phys. **28**, 1175 (1987).
- [11] Y. S. Kim and E. P. Wigner, *Space-time Geometry of Relativistic Particles*, J. Math. Phys. **31**, 55 (1990).
- [12] Y. S. Kim and M. E. Noz, *Theory and Applications of the Poincare Group*, (D. Reidel Publishing Company, 1986).